Truth tables up to three inputs

LEARNING INTENTION

We are learning to complete truth tables for logic circuits with up to three inputs.

SUCCESS CRITERIA – I CAN

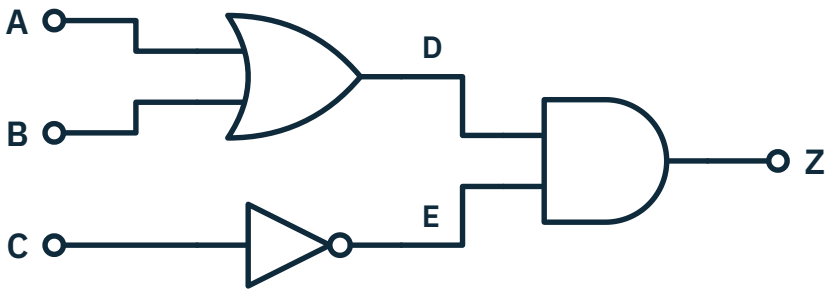
- ☐ list all input combinations in binary order
- ☐ add a column for each gate output
- ☐ work out each column from the ones before it
- ☐ complete the final output column

Listing the inputs

A truth table shows the output for **every** combination of inputs. Two inputs give 4 rows. Three inputs give 8 rows.

List the rows in binary order so none are missed.

Working through a circuit



$Z = (A + B) \cdot \overline{C}$, with intermediate outputs D and E

Add a column for each gate output (D, E). Fill them in first, then use them to find Z.

A	B	C	D	E	Z
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	1	1
1	0	1	1	0	0
1	1	0	1	1	1
1	1	1	1	0	0

Truth tables up to three inputs

WORKED EXAMPLE **Truth table for $Z = (A + B) \cdot \overline{C}$**

1 GIVEN	2 FIND	3 FORMULA	4 SUBSTITUTE	5 ANSWER + UNIT
3 inputs: A, B, C D = A + B E = NOT C	Z	8 rows: 000 to 111 work D, then E, then Z = D • E	D: 1 if A or B is 1 E: opposite of C Z: 1 only if D = 1 and E = 1	**Z = 1 for 010, 100, 110**

Count in binary down the A, B, C columns: C changes every row, B every 2, A every 4.

SHORT CIRCUIT

× WHAT EXAMINERS SAW

Rows missed or repeated because the inputs were not listed in order, so the output column was wrong.

✓ THE FIX

Always list inputs in binary order: C alternates 0,1; B in pairs; A in fours. 3 inputs = 8 rows.

Truth-table questions in recent papers; most candidates did well when columns were in order.

Reading a truth table

To describe what a circuit does, look for the rows where Z = 1. Here: Z = 1 when A or B is on, and C is off.

Column by column

FILLING A TRUTH TABLE COLUMN BY COLUMN

1

A	B	C	D	E	Z
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

1: inputs in binary order

2

A	B	C	D	E	Z
0	0	0	0		
0	0	1	0		
0	1	0	1		
0	1	1	1		
1	0	0	1		
1	0	1	1		
1	1	0	1		
1	1	1	1		

2: D = A + B

3

A	B	C	D	E	Z
0	0	0	0	1	
0	0	1	0	0	
0	1	0	1	1	
0	1	1	1	0	
1	0	0	1	1	
1	0	1	1	0	
1	1	0	1	1	
1	1	1	1	0	

3: E = NOT C

4

A	B	C	D	E	Z
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	1	1
1	0	1	1	0	0
1	1	0	1	1	1
1	1	1	1	0	0

4: Z = D • E

Truth tables up to three inputs



WORKED EXAMPLE Binary order trick

1 GIVEN	2 FIND	3 FORMULA	4 SUBSTITUTE	5 ANSWER + UNIT
3 inputs	the 8 rows	C: 0,1,0,1… B: 0,0,1,1… A: 0,0,0,0,1,1,1,1	000, 001, 010, 011, 100, 101, 110, 111	**8 rows**

Count 0 to 7 in binary.

WORKED EXAMPLE Truth table for $Z = (A \cdot \overline{B}) + C$

1 GIVEN	2 FIND	3 FORMULA	4 SUBSTITUTE	5 ANSWER + UNIT
3 inputs	Z	D = NOT B E = A • D Z = E + C	D: 1,1,0,0,1,1,0,0 E: 0,0,0,0,1,1,0,0	**Z = 0,1,0,1,1,1,0,1**

WRONG

Z written straight in, no D or E columns, two rows missing

Skipped columns and missing rows lose marks.

RIGHT

A	B	C	D	E	Z
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	1	1
1	0	1	1	0	0
1	1	0	1	1	1
1	1	1	1	0	0

Every row, every gate column.

REAL ENGINEERING · PORT ARDMORE RAMP INTERLOCK

The ramp at Port Ardmore (fictional) may only lower when a ferry is docked (A), the barrier is closed (B) and the wind is not too strong (C).

Engineers checked all eight combinations in a truth table before commissioning, so there was no combination they had not thought about.

Truth tables up to three inputs

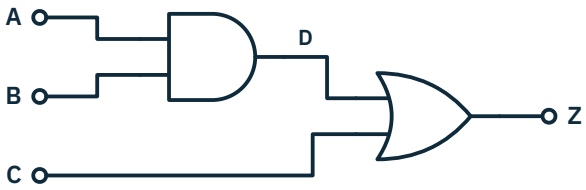
PROBE · QUICK CHECKS

1. How many rows for 3 inputs? _____
2. Which column changes every row? _____
3. Fill gate columns before or after Z? _____

EXAM READY · 6 MARKS

1 **State** the number of rows in a truth table with three inputs. **[1]**

2 **Complete** the truth table for this logic diagram. **[3]**



A	B	C	D	Z
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

3 For the circuit in Q2, **describe** when Z = 1. **[2]**