Boolean expressions

LEARNING INTENTION

We are learning to write and read Boolean expressions for logic circuits.

SUCCESS CRITERIA — I CAN

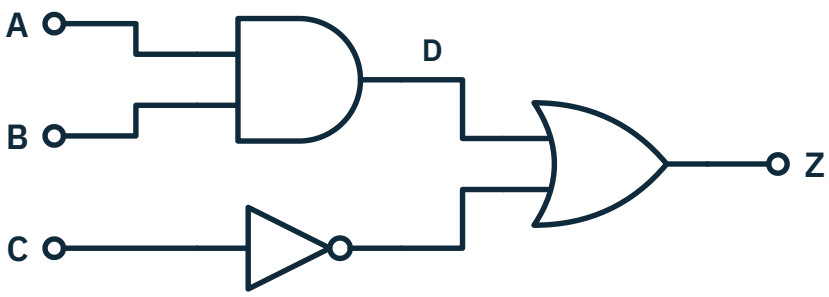
- ☐ use • for AND, + for OR and a bar for NOT
- ☐ write the expression for each gate output in turn
- ☐ use brackets to show the order of gates
- ☐ draw a logic diagram from an expression

Boolean notation

Gate	Symbol	Expression	Say it
AND	•	$Z = A \cdot B$	A and B
OR	+	$Z = A + B$	A or B
NOT	bar	$Z = \overline{A}$	not A

+ here means OR, not add. • means AND, not multiply.

From a diagram to an expression



$Z = (A \cdot B) + \overline{C}$

WORKED EXAMPLE Expression from a diagram

1 GIVEN

AND gate: inputs A, B → D
NOT gate: input C → E
OR gate: D, E → Z

2 FIND

Boolean expression for Z

3 FORMULA

$D = A \cdot B$
$E = \overline{C}$
$Z = D + E$

4 SUBSTITUTE

$Z = (A \cdot B) + \overline{C}$

5 ANSWER + UNIT

$Z = (A \cdot B) + \overline{C}$

Work one gate at a time from the inputs. Brackets show the AND happens first.

Boolean expressions

SHORT CIRCUIT

× WHAT EXAMINERS SAW

Some candidates wrote 'AND' and 'OR' as words or used × and ÷ symbols, or left out brackets, so the order of gates was unclear.

✓ THE FIX

Use the Data Booklet notation: • for AND, + for OR, bar over a letter for NOT. Bracket each gate that feeds another.

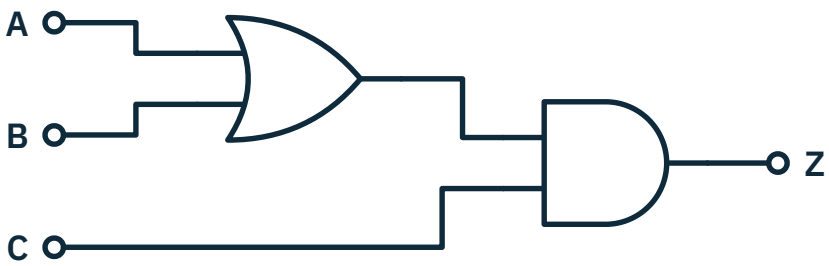
Logic questions in recent papers and course reports.

From an expression to a diagram

WORKED EXAMPLE Diagram from an expression

1 GIVEN	2 FIND	3 FORMULA	4 SUBSTITUTE	5 ANSWER + UNIT
Z = (A + B) • C	logic diagram	innermost bracket first then the outer gate	OR gate with A, B AND gate: OR output and C	**2 gates: OR then AND**

Draw the bracket first, then feed it into the gate outside the bracket.



Z = (A + B) • C

Checking with a truth table

Every expression can be checked with a truth table. Two circuits are equivalent if their output columns match.

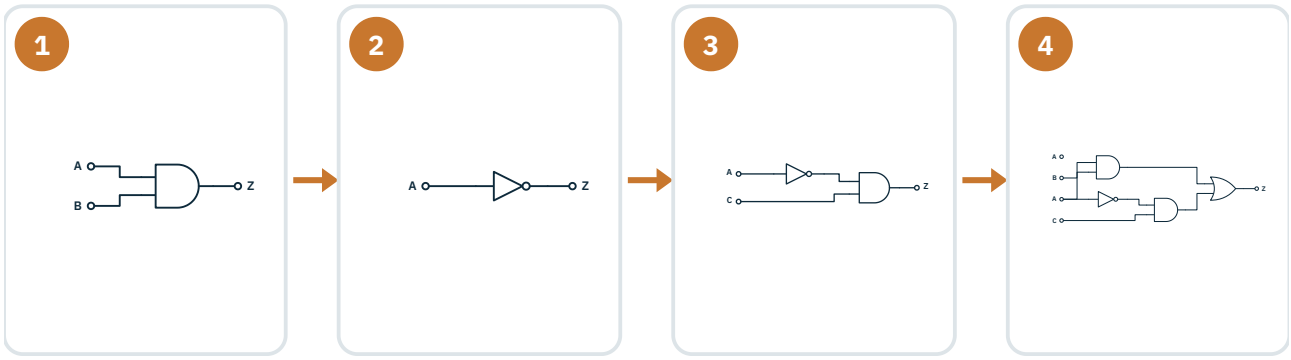
The truth table at the top of the next page checks Z = (A • B) + NOT C from page 1, with D = A • B.

Boolean expressions

A	B	C	D	Z
0	0	0	0	1
0	0	1	0	0
0	1	0	0	1
0	1	1	0	0
1	0	0	0	1
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

Building an expression gate by gate

BUILDING AN EXPRESSION GATE BY GATE



Write each gate: A • B

NOT: Ā

Ā • C

Combine: (A • B) + (Ā • C)

WORKED EXAMPLE **Reading a two-branch diagram**

1 GIVEN	2 FIND	3 FORMULA	4 SUBSTITUTE	5 ANSWER + UNIT
AND(A, B) and AND(NOT A, C) into an OR	Boolean	write each gate output then combine	top: A • B bottom: Ā • C	**Z = (A • B) + (Ā • C)**

WORKED EXAMPLE **Expression with three inputs**

1 GIVEN	2 FIND	3 FORMULA	4 SUBSTITUTE	5 ANSWER + UNIT
$Z = A \cdot \overline{B} \cdot C$	diagram	NOT on B 3-input AND	one NOT gate, one 3-input AND	**2 gates**

Boolean expressions

WRONG

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Z = A and B or not C
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Words instead of Boolean symbols lose marks.

RIGHT

$$Z = (A \cdot B) + \bar{C}$$

Use •, + and a bar, with brackets.

REAL ENGINEERING · GLENMORIE DISTILLERY STILL ALARM

The still alarm at Glenmorie (fictional) is written as $Z = (T + P) \cdot \overline{M}$: high temperature or high pressure, unless maintenance mode is on.

Writing it as an expression lets the engineer check it quickly against the safety specification.

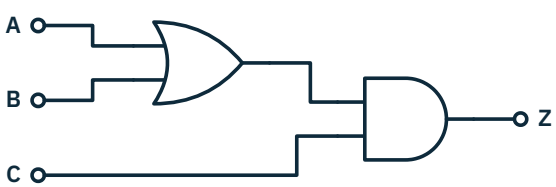
PROBE · QUICK CHECKS

1. Symbol for AND? _____
2. Symbol for OR? _____
3. How do you show NOT A? _____

EXAM READY · 5 MARKS

1 **State** the Boolean expression for a 2-input OR gate with inputs A and B. **[1]**

2 **Complete** the Boolean expression for this circuit. **[2]**



Boolean expressions

3 **Complete** the logic diagram for $Z = A \bullet \overline{B} \bullet C$. **[2]**